Pergamon

J. Appl. Maths Mechs, Vol. 58, No. 3, pp. 507–519, 1994

0021–8928/94 $24.00+0.00

0021-8928(94)00075-1

OPTIMIZATION OF THE MASS OF A WING†

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(*Received* 22 *April* 1991)

A problem of optimizing the mass of a wing is formulated for a possible permeable wing made up of a system of feathers. The basic characteristics of the external shape of the wing are assumed to be specified. An approximate solution to the problem is found. Formulae for obtaining the optimum wing mass are obtained.

1. CONCEPTS USED

A first-order feather is a system of beams, one of which is selected to have all the remaining beams cantilevered from it, with the axes of the cantilevered beams lying in a single surface. The surface containing the axes of the cantilevered beams is called a first-order feather surface, and the selected beam to which the cantilevered beams are attached is the spine of the first-order feather. The cantilevered beams themselves are sometimes called zeroth-order feathers (degenerate feathers).

For $k = 2, 3, \ldots$ a kth order feather is a system consisting of a beam and of $(k-1)$th order feathers, each of which is cantilevered from the beam by means of the spine, so that the surfaces of the $(k-1)$th order feathers coincide. The common surface of the $(k-1)$th order feathers is called the surface of the kth order feather, and the beam to which the $(k-1)$th order feathers are attached is the spine of the kth order feather. A kth order subfeather is a system consisting of a beam and $(k-1)$th order feathers cantilevered from it by means of the spines so that not all the surfaces of the $(k-1)$th order feathers coincide. The spine of a kth order feather (or subfeather) is for brevity often called a kth order spine. If n is the highest order of the feathers (or subfeathers) constituting a wing, then any surface of these feathers (or subfeathers) is called the wing surface.

A feather (or subfeather) is said to be symmetrically loaded if its spine is in a perpendicularly stressed state [1]. Here we consider spines of annular section. To calculate the dimensions of the section we will use the familiar strength condition for a cantilever spine under the action of a distributed transverse load, $4MR\pi^{-1}(R^4-r^4)^{-1} \leqslant \sigma$, where M is the bending moment acting in a section of the spine, R is the outer radius of the section, r is the inner radius of the section, and σ is the breaking stress. Using this, the smallest cross-sectional area for a given R and M is described by the function

$$F(R,M)=\begin{cases}\pi R^2\left(1-\sqrt{1-4M/(\pi\sigma R^3)}\right) \text{ when } \pi\sigma R^3 \geqslant 4M\\ \infty \text{ – otherwise}\end{cases} \tag{1.1}$$

†*Prikl. Mat. Mekh.* Vol. 58, No. 3, pp. 131–143, 1994.

2. FORMULATION OF THE PROBLEM

In order to describe the shape of the wing in plan we will introduce a skewed Cartesian system of coordinates x, y with an angle φ_0 between the x and y axes, assuming that $0<\varphi_0<\pi$. Suppose that this shape is a parallelogram bounded by the coordinate lines $x=0$, $x=b$, $y=0$, $y=l$, where b and l are given positive numbers. The wing is attached at the $y=0$ side. We will consider two cases of wing construction: (1) with a single surface common to the nth order feathers, and (2) with two surfaces common to nth order subfeathers (n is variable). Wing sections for the first and second cases are shown in Figs 1(a) and 1(b).

We will formulate the problem for the first case, taking the wing surface to be the $z=0$ plane (the z axis being perpendicular to the x and y axes), and then point out the properties of the second case.

The plan shape of the wing when $n=1$ is shown in Fig. 2. We shall assume that the pressure drop through the surface of the wing at any point $(x, y, 0)$ on this surface is given by the product $p_1(x)p_2(y)$ for $x\in[0, b]$, $y\in[0, l]$, where p_1 and p_2 are known continuous functions, positive inside the specified intervals. We shall later show how the force of gravity can be taken into account, but for the time being we shall assume that it can be neglected.

Taking the projection of a wing (or a feather) to be its projection onto the $z=0$ plane, we shall consider versions of wings in the form of collections of nth order feathers which satisfy the following restrictions: (a) the wing projection decomposes into projections of nth order feathers with the help of the lines $x=\text{const}$, (b) the axis of the spine of a kth order feather ($k=n, n-1, \ldots, 0$) is an interval of the line $x=\text{const}$ when $(n-k)$ is even and of the line $y=\text{const}$ otherwise, the position of the axis being chosen so that the feather is symmetrically loaded, (c) the spine axis of a kth order feather ($k=n, n-1, \ldots, 1$) divides the projection of this feather into subprojections which decompose into the projections of $(k-1)$th order feathers with the help of the lines $x=\text{const}$ when $(n-k)$ is odd, and $y=\text{const}$ otherwise, where the decomposition of a subprojection when $k>1$ is performed so that the spines of the $(k-1)$th order feathers have identical loads, (d) the external radius of the section of an nth order spine at any point (x, y) on the axis is given by a specified function $f(y)$ ($y\in[0, l]$), which

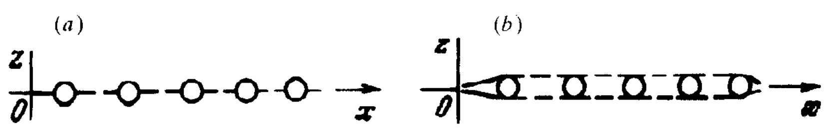

Fig. 1.

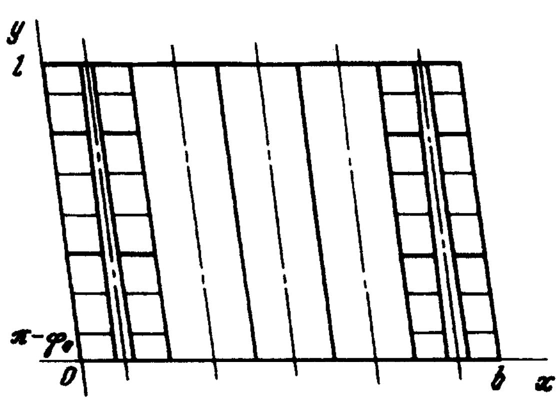

Fig. 2.

is concave, positive and decreasing, (e) the external radius of the section of a kth order spine, where $0 \leqslant k \leqslant n-1$, is bounded by the values $f(l)$ when $1 \leqslant k \leqslant n-1$, and δ when $k=0$, where the parameter δ is imposed by quality requirements on the surface, $\delta \leqslant f(l)$.

We introduce the fundamental variables. First, we introduce the variable m, which is the number of nth order feathers in the wing, and relate it to the mass of the nth order spines.

By hypothesis, the projection of the wing decomposes into the projections of nth order feathers with the help of the lines

$$x = x_{i_1}(m), \quad i_1 = 1, \ldots, m+1 \tag{2.1}$$

where the parameters are found from the condition of equal spine loading

$$\int\limits_{x_{i_1}(m)}^{x_{i_1+1}(m)} p_1(x)dx = \frac{1}{m}\int\limits_0^b p_1(x)dx, \quad i_1 = 1, \ldots, m \tag{2.2}$$

where $x_1(m) = 0$. The i_1th and (i_1+1)th pair of adjacent lines from (2.1) together with the $y=0$ and $y=1$ lines bound the projection of the i_1th feather of nth order. The spine axis of this feather is the section of the line $x = x^c_{i_1}(m)$, bounded by the lines $y=0$ and $y=l$, where $x = x^c_{i_1}(m)$, is found from the condition for symmetrical loading of the feather

$$\int\limits_{x_{i_1}(m)}^{x^c_{i_1}(m)} p_1(x)[x^c_{i_1}(m) - x]dx = \int\limits_{x^c_{i_1}(m)}^{x_{i_1+1}(m)} p_1(x)[x - x^c_{i_1}(m)]dx \tag{2.3}$$

The spine is secured in the section where $y=0$, so that in the section appropriate to some point $(x^c_{i_1}(m), y)$ of the axis there is obviously a bending moment

$$M_{i_1}(m,y) = \sin\varphi_0 \int\limits_{x_{i_1}(m)}^{x_{i_1+1}(m)} p_1(x)dx \int\limits_y^l p_2(\bar{y})(\bar{y} - y)d\bar{y}$$

The smallest cross-sectional area corresponding to this moment is $F[f(y), M_{i_1}(m, y)]$, where F is a function of the form (1.1). Taking this into account, we conclude that the spine of the i_1th feather of nth order is of mass

$$G_{i_1}(m) = \rho\int\limits_0^l F[f(y), M_{i_1}(m,y)]dy$$

where ρ is the density of the wing material. We shall impose a restriction on the variable m with respect to the ratio of the spine radius and the size of the feather projection

$$\sin\varphi_0 |x^c_{i_1}(m) - x_{i_1+j}(m)| > wf(0), \quad i_1 = 1, \ldots, m; \quad j = 0,1$$

where the number w is assumed to be specified so that $w \geqslant 1$.

We have thus introduced the variable m and related it to the mass of the spines of the nth order feathers. Then, for $i_1 = 1, \ldots, m$ we introduce a variable m_{i_1}, which is the number of pairs of $(n-1)$th order feathers constituting the nth order feather with number i_1. We relate the variables m_{i_1} to the mass of the $(n-1)$th order spines.

The spine axis of the i_1th feature of nth order decomposes the projection of this feather into two subprojections, which are numbered by the index j_1. The value $j_1 = 0$ corresponds to the subprojection at whose points $x \leqslant x^c_{ij}(m)$, while the value $j_1 = 1$ corresponds to the other subprojection. By hypothesis, each of the subprojections is divided up by the lines

$$y = y_{i_2}(m_{i_1}), \quad i_2 = 1, \ldots, m_{i_1} + 1 \tag{2.4}$$

into projections of $(n-1)$th order feathers, the lines satisfying the conditions of equal spine loading

$$\int_{y_{i_2}(m_{i_1})}^{y_{i_2+1}(m_{i_1})} p_2(y)dy = \frac{1}{m_{i_1}}\int_0^l p_2(y)dy, \quad i_2 = 1,\dots,m_{i_1}$$

with $y_1(m_{i_1})=0$. Thus any adjacent i_2th and (i_2+1)th lines from (2.4), together with the lines

$$x = x_{i_1+j_1}(m), \quad x = x^c_{i_1}(m) \tag{2.5}$$

bound the projection of the $(n-1)$th order feather with number (i_1, i_2, j_1). The spine axis of this feather is the section of the line $y = y^c_{i_2}(m_{i_1})$ enclosed by the lines (2.5), where the parameter of the axis is found from the condition for symmetric loading of the feather

$$\int_{y_{i_2}(m_{i_1})}^{y^c_{i_2}(m_{i_1})} p_2(y)[y^c_{i_2}(m_{i_1})-y]dy = \int_{y^c_{i_2}(m_{i_1})}^{y_{i_2+1}(m_{i_1})} p_2(y)[y-y^c_{i_2}(m_{i_1})]dy$$

We shall provisionally assume that the spine is attached in the section where $x = x^c_{i_2}(m)$, so that in a section corresponding to some point $(x, y^c_{i_2}(m_{i_1}))$ of the axis there is a bending moment

$$M^{j_1}_{i_1 i_2}(m, m_{i_1}; x) = \sin\varphi_0 \int_{x_{i_1+j_1}(m)}^{x} p_1(\bar{x})(x-\bar{x})d\bar{x} \int_{y_{i_2}(m_{i_1})}^{y_{i_2+1}(m_{i_1})} p_2(y)dy$$

Taking the outer radius of the section to have the largest admissible value $f(l)$ (minimizing the area of the section), we express the mass of the spine numbered (i_1, i_2, j_1) in the form

$$G^{j_1}_{i_1 i_2}(m, m_{i_1}) = (-1)^{j_1}\rho \int_{x_{i_1+j_1}(m)}^{x^c_{i_1}(m)} F[f(l), M^{j_1}_{i_1 i_2}(m, m_{i_1}; x)]dx$$

The variable m_{i_1} is restricted by the sizes of the feather projection and spine radius

$$\sin\varphi_0 |y^c_{i_2}(m_{i_1}) - y_{i_2+j}(m_{i_1})| \geq f(l), \quad i_2 = 1,\dots,m_{i_1}; \quad j = 0,1$$

Thus for feathers of order $k = n-1$ we have introduced variables m_{i_1} $(i_1 = 1, \dots, m)$ which we have related to the mass of the spines of those feathers. We perform a similar formalization for feathers of order $k = n-2, \dots, 1$. In particular, if we have performed the formalization for $k = n-s$, where $1 \leq s < n-1$, then for $k = n-s-1$ we proceed as follows.

We introduce the variable $m^{j(s)}_{i(s+1)}$ which is the number of pairs of $(n-s-1)$th order feathers in the $(n-s)$th order feather with number $(i(s+1), j(s))$. Here and below the index $j(s)$ denotes the set of indices $j_1, \dots, j_s$, and the index $i(s)$ denotes the set of indices $i_1, \dots, i_s$. In the newly-introduced variable the indices take the following values

$$i_1 = 1,\dots,m; \quad i_2 = 1,\dots,m_{i_1}; \ \dots \ ; \ i_{s+1} = 1,\dots,m^{j(s-1)}_{i(s)}; \quad j_1 = 0,1; \ \dots; \ j_s = 0,1$$

To simplify the formalism we introduce the following notation

$$\mu^{j(s)}_{i(s+1)} = \begin{cases} (m, m^{j(1)}_{i(2)}, \dots, m^{j(s)}_{i(s+1)}) & \text{with odd} \\ (m_{i_1}, m^{j(2)}_{i(3)}, \dots, m^{j(s)}_{i(s+1)}) & \text{with even} \end{cases}$$

Two cases are possible: (1) s odd, and (2) s even. We shall only consider the first case, because the formulae for the second case are similar, the main difference being that the x and y coordinates and the functions $p_1(x)$ and $p_2(x)$ are interchanged. The spine axis of the feather numbered $(i(s+1), j(s))$, where s is odd, decomposes the projection of this feather into two subprojections which we number with the index j_{s+1}. $j_{s+1} = 0$ corresponds to the subprojection at whose points $y \leq y^c_{i_{s+1}}(\mu^{j(s-1)}_{i(s)})$, and $j_{s+1} = 1$ to the other subprojection. By assumption, each of the subprojections is divided into projections of $(n-s-1)$th order feathers by the lines

$$x = x_{i_{s+2}}(\mu_{i(s+1)}^{j(s)}), \quad i_{s+2} = 1,\dots,m_{i(s+1)}^{j(s)} + 1 \tag{2.6}$$

satisfying the conditions for equal loading of the spines

$$\int_{x_{i_{s+2}}(\mu_{i(s+1)}^{j(s)})}^{x_{i_{s+2}+1}(\mu_{i(s+1)}^{j(s)})} p_1(x)dx = \int_{x_{i_s+j_s}(\mu_{i(s-1)}^{j(s-2)})}^{x_{i_s}^c(\mu_{i(s-1)}^{j(s-2)})} p_1(x)dx(-1)^{j_s} / m_{i(s+1)}^{j(s)}, \quad i_{s+2} = 1,\dots,m_{i(s+1)}^{j(s)}$$

where

$$x_1(\mu_{i(s+1)}^{j(s)}) = \begin{cases} x_{i_s}(\mu_{i(s-1)}^{j(s-2)}) & \text{when } j_s = 0 \\ x_{i_s}^c(\mu_{i(s-1)}^{j(s-2)}) & \text{when } j_s = 1 \end{cases}$$

A pair of adjacent lines from (2.6) numbered i_{s+2} and $(i_{s+2}+1)$, together with the lines

$$y = y_{i_{s+1}+j_{s+1}}(\mu_{i(s)}^{j(s-1)}), \quad y = y_{i_{s+1}}^c(\mu_{i(s)}^{j(s-1)}) \tag{2.7}$$

bound the projection of the $(n-s-1)$th order feather numbered $(i(s+2),\ j(s+1))$. The spine axis of this feather is the section of the line $x = x_{i\ s+2}^c(\mu_{i(s+1)}^{j(s)})$, enclosed by the lines (2.7), and we find the axis parameter from the condition for symmetric loading of the feather

$$\int_{x_{i_{s+2}}(\mu_{i(s+1)}^{j(s)})}^{x_{i_{s+2}}^c(\mu_{i(s+1)}^{j(s)})} p_1(x)[x_{i_{s+2}}^c(\mu_{i(s+1)}^{j(s)}) - x]dx = \int_{x_{i_{s+2}}^c(\mu_{i(s+1)}^{j(s)})}^{x_{i_{s+2}+1}(\mu_{i(s+1)}^{j(s)})} p_1(x)[x - x_{i_{s+2}}^c(\mu_{i(s+1)}^{j(s)})]dx$$

In the section corresponding to any point $(x = x_{i\ s+2}^c(\mu_{i(s+1)}^{j(s)}),\ \ y)$ of the spine axis there is a bending moment

$$M_{i(s+2)}^{j(s+1)}(m, m_{i_1},\dots,m_{i(s+1)}^{j(s)}; y) = \sin\varphi_0 \int_{x_{i_{s+2}}(\mu_{i(s+1)}^{j(s)})}^{x_{i_{s+2}+1}(\mu_{i(s+1)}^{j(s)})} p_1(x)dx \int_{y_{i_{s+1}+j_{s+1}}(\mu_{i(s)}^{j(s-1)})}^{y} p_2(\bar{y})(y-\bar{y})d\bar{y} \tag{2.8}$$

Taking the outer radius of the section as the largest permissible value $f(l)$, we write down the mass of the spine for the feather numbered $(i(s+2),\ j(s+1))$

$$G_{i(s+2)}^{j(s+1)}(m, m_{i_1},\dots,m_{i(s+1)}^{j(s)}) = \\ = (-1)^{j_{s+1}}\rho \int_{y_{i_{s+1}+j_{s+1}}(\mu_{i(s)}^{j(s-1)})}^{y_{i_{s+1}}^c(\mu_{i(s)}^{j(s-1)})} F[f(l), M_{i(s+2)}^{j(s+1)}(m, m_{i_1},\dots, m_{i(s+1)}^{j(s)}; y)]dy \tag{2.9}$$

The variable $m_{i(s+1)}^{j(s)}$ should satisfy the restriction

$$\sin\varphi_0 |x_{i_{s+2}}^c(\mu_{i(s+1)}^{j(s)}) - x_{i_{s+2}+\gamma}(\mu_{i(s+1)}^{j(s)})| \geq f(l) \tag{2.10}$$

$$i_{s+2} = 1,\dots,m_{i(s+1)}^{j(s)}; \quad \gamma = 0, 1$$

We have thus performed the formalization for feathers of order $k = n-s-1$, when $s < n-1$. For $s = n-1$, i.e. for feathers of order $k = 0$, the formalization is similar to that described, although there are differences. We will indicate these differences (for the case of odd s) as follows.

1. The parameters of the lines (2.6) are determined from the condition for the external spine

diameters to be equal, i.e. from the condition

$$x_{i_{s+2}+1}(\mu^{j(s)}_{i(s+1)}) - x_{i_{s+2}}(\mu^{j(s)}_{i(s+1)}) = 2r(\mu^{j(s)}_{i(s+1)}) / \sin\varphi_0$$

where

$$r(\mu^{j(s)}_{i(s+1)}) = \frac{(-1)^{j_s}[x^c_{i_s}(\mu^{j(s-2)}_{i(s-1)}) - x_{i_s+j_s}(\mu^{j(s-2)}_{i(s-1)})]}{2m^{j(s)}_{i(s+1)}}\sin\varphi_0$$

is the outer radius of the spine.

2. The spine axis parameter $x^c_{i_{s+2}}(\mu^{j(s)}_{i(s+1)})$ is the average of the parameters $x_{i_{s+2}+\gamma}(\mu^{j(s)}_{i(s+1)})$, $\gamma = 0, 1$.

3. In (2.9) $f(l)$ is replaced by $r(\mu^{j(s)}_{i(s+1)})$.

4. Instead of restriction (2.10) the surface quality restriction $r(\mu^{j(s)}_{i(s+1)}) \leqslant \delta$ is used. We will finish the formalization by writing out the efficiency criterion

$$\sum_{i_1=1}^{m}\left[G_{i_1}(m) + \sum_{j_1=0}^{1}\sum_{i_2=1}^{m_{i(1)}} G^{j(1)}_{i(2)}(m, m_{i_1}) + \ldots + \sum_{j_1=0}^{1}\sum_{i_2=1}^{m_{i(1)}} \ldots \sum_{j_n=0}^{1}\sum_{i_{n+1}=1}^{m^{j(n-1)}_{i(n)}} G^{j(n)}_{i(n+1)}(m, m_{i_1}, \ldots, m^{j(n-1)}_{i(n)})\right]$$

It is required to minimize this criteria with respect to the variables n, m, m_{i_1}, $m^{j(1)}_{i(2)}, \ldots, m^{j(n-1)}_{i(n)}$ under the given conditions.

3. SOLUTION OF THE PROBLEM

Suppose $n > 2$. We shall first show that in order to reduce the value of the efficiency criterion it is desirable to increase the value of each variable $m^{j(n-2)}_{i(n-1)}$, describing the number of first-order feathers in the second-order feather numbered $(i(n-1), j(n-2))$. To do this we shall prove two assertions. The first shows that when $m^{j(n-2)}_{i(n-1)}$ increases, the total mass of the first-order feather spines decreases in the second-order feather numbered $(i(n-1), j(n-2))$. It follows from the second assertion that when the same variable is increased ν times ($\nu = 2, 3, \ldots$) there is a reduction in the total mass of the zeroth-order spines entering into the same second-order feather.

Assertion 1. Suppose there are two sets of symmetrically loaded feathers: Π^0_i $(i = 1, \ldots, \nu_0)$ and Π^1_i $(i = 1, \ldots, \nu_1)$ and that for each feather $\Pi^\kappa_i(\kappa = 0, 1;\ i = 1, \ldots, \nu_\kappa)$ the spine axis is the section of the line $\xi = \xi^\kappa_i$ enclosed between the lines $\eta = 0$ and $\eta = a$ in the Cartesian system of coordinates $O\xi\eta$, $a > 0$. Suppose that the spine section through some point (ξ^κ_i, η) of the axis has an outer radius $R(\eta) > 0$ $(\eta \in [0, a])$, and that in this section there is a bending moment $M^\kappa_i(\eta)$ the internal radius of the section being chosen so that the area of the section is the smallest one preserving the rigidity of the spine.

We introduce the notation

$$\Sigma^s_\kappa(\eta) = \sum_{i=1}^{\nu_\kappa}[M^\kappa_i(\eta)]^s$$

Suppose, further, that the functions $R(\eta)$ and $M^\kappa_i(\eta)(\eta \in [0, a])$ are continuous, and that

$$M^\kappa_i(\eta) > 0, \quad \eta \in [0, a], \quad \kappa = 0, 1; \quad i = 1, \ldots, \nu_\kappa \tag{3.1}$$

$$\Sigma^1_0(\eta) = \Sigma^1_1(\eta), \quad \eta \in [0, a] \tag{3.2}$$

$$M^\kappa_i(\eta) = M^\kappa_j(\eta), \quad \eta \in [0, a], \quad \kappa = 0, 1; \quad i, j = 1, \ldots, \nu_\kappa \tag{3.3}$$

Then, if $\nu_0 < \nu_1$, the total mass of the spines of the Π^0_i $(i = 1, \ldots, \nu_0)$ feathers is greater than the total mass of the spines of the Π_i^1 $(i=1, \ldots, \nu_1)$ feathers.

Proof. From the conditions of the assertion, the area of the spine sections for each feather Π_i^κ $(\kappa=0, 1;$ $i=1, \ldots, \nu_\kappa)$ is described by the continuous function

$$F[R(\eta), M_i^\kappa(\eta)] = \pi[R(\eta)]^2[1-\sqrt{1-M_i^\kappa(\eta)\alpha(\eta)}], \quad \eta \in [0, a]$$

$$\alpha(\eta) = 4(\pi\sigma)^{-1}[R(\eta)]^{-3} > 0$$

where the expression under the root is non-negative. The volume of the spine of the Π_i^κ feather is equal to the integral of $F[R(\eta), M_i^\kappa(\eta)]$ over the interval $[0, a]$. In order to prove the assertion it is therefore sufficient to show that

$$\sum_{i=1}^{\nu_0} F[R(\eta), M_i^0(\eta)] > \sum_{i=1}^{\nu_1} F[R(\eta), M_i^1(\eta)], \quad \eta \in [0, a)$$

Expanding the function F in series, we can write this inequality in the form

$$\frac{\alpha(\eta)}{2}\Sigma_0^1(\eta) + \frac{\alpha^2(\eta)}{2\cdot 4}\Sigma_0^2(\eta) + \ldots > \frac{\alpha(\eta)}{2}\Sigma_1^1(\eta) + \frac{\alpha^2(\eta)}{2\cdot 4}\Sigma_1^2(\eta) + \ldots \tag{3.4}$$

We use the fact that when $\nu_0 < \nu_1$ the relation

$$\Sigma_0^s(\eta) > \Sigma_1^s(\eta), \quad \eta \in [0, a), \quad s = 2, 3, \ldots$$

is satisfied by virtue of conditions (3.1)–(3.3).

The validity of (3.4) then follows from the series comparison theorem.

Remark. If we have $\nu_0 = 1$ in the formulation of Assertion 1 (one feather being replaced by ν_1 feathers), then the assertion still holds without conditions (3.3).

Assertion 2. Suppose we have a pair of spines C_i^0 $(i=1, \ldots, 2\nu_0)$ and ν_1 pairs of spines C_i^1 $(i=1, \ldots, 2\nu_1)$, where $\nu_0 = 1 < \nu_1$ (Fig. 3), where the axis of any spine C_i^κ $(\kappa=0, 1;$ $i=1, \ldots, 2\nu_1)$, is described by a section $[\xi_{i-1}^\kappa, \xi_i^\kappa]$ of the ξ coordinate axis such that

$$\xi_0^\kappa < \xi_1^\kappa < \ldots < \xi_{2\nu_\kappa}^\kappa \ (\kappa = 0, 1), \quad \xi_0^0 = \xi_0^1, \quad \xi_2^0 = \xi_{2\nu_1}^1 \tag{3.5}$$

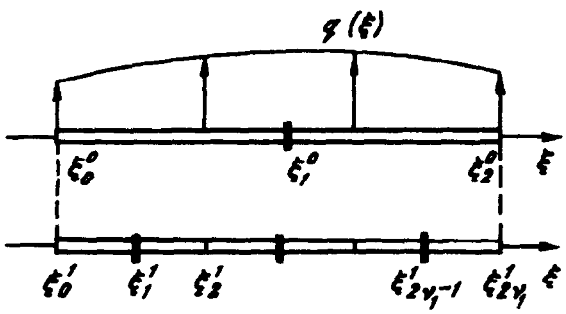

Fig. 3.

Furthermore, we will specify a continuous function $q(\xi)$, $\xi \in [\xi_0^0, \xi_2^0]$ whose restriction to any interval $[\xi_{i-1}^\kappa, \xi_i^\kappa]$ gives the distribution of the transverse load acting on the spine C_i^κ, and $q(\xi) > 0$ when $\xi_0^0 < \xi < \xi_2^0$. Suppose that the spines C_{2s-1}^κ and C_{2s}^κ, where $1 \leq s \leq \nu_\kappa$, are fixed in a neighbourhood of the points ξ_{2s-1}^κ, and that in sections of the spine near this point the bending moments are equal

$$\int_{\xi_{2(s-1)}^\kappa}^{\xi_{2s-1}^\kappa} q(\xi)(\xi_{2s-1}^\kappa - \xi)d\xi = \int_{\xi_{2s-1}^\kappa}^{\xi_{2s}^\kappa} q(\xi)(\xi - \xi_{2s-1}^\kappa)d\xi \quad (s = 1, \ldots, \nu_\kappa)$$

Finally, let the outer radius of the section of each spine be $R > 0$, and the inner radius be chosen so that the cross-sectional area is the smallest consistent with the structural integrity of the spine. Then the mass of the pair of spines C_i^0 $(i = 1, 2)$ is greater than the total mass of the ν_1 pairs of spines C_i^0 $(i = 1, \ldots, 2\nu_1)$.

Proof. The bending moments acting in different sections of the spine c_{2s-1+j}^κ $(\kappa = 0, 1; s = 1, \ldots, \nu_\kappa; j = 0, 1)$, are described by a function

$$M_{2s-1+j}^\kappa(\xi) = \int_{\xi_{2(s-1+j)}^\kappa}^{\xi} q(\bar{\xi})(\xi - \bar{\xi})d\bar{\xi}, \quad \xi \in [\xi_{2(s-1)+j}^\kappa, \xi_{2s-1+j}^\kappa] \tag{3.6}$$

that is continuous and monotonic (using the properties of q). In addition, from the conditions of the assertion the relations

$$M_i^\kappa(\xi_i^\kappa) = M_{i+1}^\kappa(\xi_i^\kappa) \quad (\kappa = 0,1; \ i = 1, \ldots, 2\nu_\kappa - 1) \tag{3.7}$$

hold.

Hence for $\kappa = 0, 1$ one can construct the continuous function

$$M^\kappa(\xi) = M_i^\kappa(\xi), \quad \xi \in [\xi_{i-1}^\kappa, \xi_i^\kappa], \quad i = 1, \ldots, 2\nu_\kappa \tag{3.8}$$

and express the volume of each set of spines C_i^κ $(i = 1, \ldots, 2\nu_\kappa)$ as an integral of the function

$$F(R, M^\kappa(\xi)) = \pi R^2[1 - \sqrt{1 - 4M^\kappa(\xi)(\pi\sigma R^3)^{-1}}], \quad \xi \in [\xi_0^0, \xi_2^0]$$

where the expression under the square root sign is non-negative. Since the function $F(R, M^\kappa(\xi))$ increases with $M^\kappa(\xi))$ then, from the properties of the integral, to complete the proof it is sufficient to justify the inequality $M^0(\xi) \geq M^1(\xi)$ when $\xi \in [\xi_0^0, \xi_2^0]$, and in addition, to show that this inequality is strict on some interval $[\xi', \xi'']$ where $\xi_0^0 \leq \xi' < \xi'' \leq \xi_2^0$. To this end we take any $s \in \{1, \ldots, \nu_1\}$ and consider the relation between $M^0(\xi)$ and $M^1(\xi)$ in the interval $[\xi_{2(s-1)}^1, \xi_{2s}^1]$.

Two cases are possible: (1) $\xi_{2s-1}^1 < \xi_1^0$ and (2) $\xi_{2s-1}^1 \geq \xi_1^0$. The arguments are similar in the two cases, so we shall only consider the first. In that case, using (3.8), (3.6), (3.5) and the properties of q, we have

$$M^0(\xi) = M_1^0(\xi) \geq M_{2s-1}^1(\xi) = M^1(\xi), \quad \xi \in [\xi_{2(s-1)}^1, \xi_{2s-1}^1] \tag{3.9}$$

We consider the relation between $M^0(\xi)$ and $M^1(\xi)$ in the interval $(\xi_{2s-1}^1, \xi_{2s}^1]$. The possible situations are: (a) $\xi_{2s}^1 > \xi_1^0$ and (b) $\xi_{2s}^1 \leq \xi_1^0$. We shall only consider the first of these, the proof for the second case being similar. In case (a), assuming that $\xi_{2s}^1 = \xi_2^0$, we obtain the relation

$$M_{2s-1}^1(\xi_{2s-1}^1) < M_1^0(\xi_1^0) = M_2^0(\xi_1^0) < M_{2s}^1(\xi_{2s-1}^1)$$

contradicting (3.7). Hence $\xi_{2s}^1 < \xi_2^0$ and like (3.9) we have

$$M^0(\xi) = M_2^0(\xi) > M_{2s}^1(\xi) = M^1(\xi), \quad \xi \in [\xi_1^0, \xi_{2s}^1]$$

Moreover, using (3.6)–(3.9), we obtain

$$M^0(\xi)=M_1^0(\xi)>M_1^0(\xi_{2s-1}^1)\geqslant M_{2s-1}^1(\xi_{2s-1}^1)=M_{2s}^1(\xi_{2s-1}^1)>M_{2s}^1(\xi)=M^1(\xi),\quad \xi\in(\xi_{2s-1}^1,\xi_1^0]$$

The assertion is proved.

It follows from Assertions 1 and 2 that it is desirable to assign the smallest value allowed by restrictions (2.10) to the variable $m_{i(n-1)}^{j(n-2)}$, i.e. to take

$$m_{i(n-1)}^{j(n-2)}=\overline{m}_{i(n-1)}^{j(n-2)}(\mu_{i(n-3)}^{j(n-4)})\tag{3.10}$$
$$(i_1=1,\ldots,m;\ldots;i_{n-1}=1,\ldots,m_{i(n-2)}^{j(n-3)};\; j_1=0,1;\ldots;j_{n-2}=0,1)$$

since this will minimize (for the specified variable) the total mass of the first-order spines, and moreover, this turns out to be close to the smallest total mass of the zeroth-order spines, and at the same time preserves the non-varying mass of the spines of order 2, . . . , n. From the same assertions it also follows that it is desirable to assign to the variables

$$m_{i(n-2)}^{j(n-3)}\quad(i_1=1,\ldots,m;\ldots;i_{n-2}=1,\ldots,m_{i(n-3)}^{j(n-4)};\; j_1=0,1;\ldots;j_{n-3}=0,1)$$

the largest possible values compatible with conditions (3.10). Here we will achieve the minimum total mass of the second order spines (with respect to the indicated variables), it is close to the minimum total mass for the first-order and second-order spines, and the masses of spines of orders 3, . . . , n are unchanged. The length of each first-order spine is near to the value $f(l)$ of the outer radius of a second-order spine. In this case such degenerate first-order spines can be eliminated from the make-up of the wing without significant loss of mass, i.e. n can be reduced by 1.

Continuing the argument, we conclude that to minimize the efficiency criterion it is desirable to take $n=2$, $m=\overline{m}$, $m_{i_1}=\overline{m}_{i_1}$ $(i_1=1,\ldots,m)$, where the largest allowed value of the variable is noted. The smallest possible value of $m_{i_1}^{j_1}$, which maximizes the outer radius of the zeroth-order spine section and minimizes its mass, is assigned each variable $m_{i(2)}^{j(1)}$ $(i_1=1,\ldots,$ m; $i_2=1,\ldots,m_{i_1}$; $j_1=0, 1)$, giving the number of zeroth-order spine pairs in the first-order feather with number (i_1, i_2, j_1). The possible error in the minimum efficiency criterion when these values are used is smaller than the total mass of the zeroth-order and first-order spines.

We shall estimate the total mass of the spines for each order $k=0, 1, 2$. We first obtain an estimate for $k=0$, taking $s=1$ in expressions (2.8) and (2.9), and replacing $f(l)$ by $r(\mu_{i(2)}^{j(1)})$ in (2.9). We take into account that the bigger the variables m, m_{i_1}, $m_{i(2)}^{j(1)}$, the closer the limits of integration in each integral of formulae (2.8) and the smaller the moment (2.8). We assume that this moment is sufficiently small that one can neglect the error caused by replacing the function F with the first term of its series expansion in (2.9). Then

$$G_{i(3)}^{j(2)}(m,m_{i_1},m_{i(2)}^{j(1)})\approx\frac{\beta}{r(\mu_{i(2)}^{j(1)})}\int\limits_{x_{i_3}(\mu_{i(2)}^{j(1)})}^{x_{i_3+1}(\mu_{i(2)}^{j(1)})}p_1(x)dx(-1)^{j_2}\int\limits_{y_{i_2+j_2}(m_{i_1})}^{y_{i_2}^c(m_{i_1})}dy\int\limits_{y_{i_2+j_2}(m_{i_1})}^{y}p_2(\bar y)(y-\bar y)d\bar y=$$
$$=\frac{\beta p_2(y_{i_2}^{j_2})[\Delta y_{i_2}^{j_2}(m_{i_1})]^3}{6r(\mu_{i(2)}^{j(1)})}\int\limits_{x_{i_3}(\mu_{i(2)}^{j(1)})}^{x_{i_3+1}(\mu_{i(2)}^{j(1)})}p_1(x)dx$$

where

$$\beta=2\rho\sin\varphi_0/\sigma,\quad \Delta y_{i_2}^{j_2}(m_{i_1})=(-1)^{j_2}[y_{i_2}^c(m_{i_1})-y_{i_2+j_2}(m_{i_1})]$$

and the number $y_{i_2}^{j_2}$, which ensures that the equality is satisfied, lies between $y_{i_2+j_2}(m_{i_1})$ and $y_{i_2}^c(m_{i_1})$, by the mean value theorem. We introduce the following notation

$$l_*(\mu^0)=\max_{i_1,i_2,j_2}\Delta y_{i_2}^{j_2}(m_{i_1}),\quad \mu^0=\{m_{i_1}\}$$

$$r_*(\mu^1) = \min_{i_1, i_2, j_1} r(\mu_{i(2)}^{j(1)}), \quad \mu^1 = \{m_{i(2)}^{j(1)}\}$$

Using this notation and the definition of the integral, we obtain

$$\sum_{i_1=1}^{m} \sum_{j_1=0}^{1} \sum_{i_2=1}^{m_{i(1)}} \sum_{j_2=0}^{1} \sum_{i_3=1}^{m_{i(2)}^{j(1)}} G_{i(3)}^{j(2)}(m, m_{i_1}, m_{i(2)}^{j(1)}) \approx$$

$$\approx \sum_{i_1, j_1} \sum_{i_2, j_2} \frac{(-1)^{j_1} \beta p_2(y_{i_2}^{j_2})[\Delta y_{i_2}^{j_2}(m_{i_1})]^3}{6r(\mu_{i(2)}^{j(1)})} \int_{x_{i_1+j_1}(m)}^{x_{i_1}^c(m)} p_1(x)dx \leqslant$$

$$\leqslant \frac{\beta l_*^2(\mu^0)}{6r_*(\mu^1)} \sum_{i_1, j_1} (-1)^{j_1} \int_{x_{i_1+j_1}(m)}^{x_{i_1}^c(m)} p_1(x)dx \sum_{i_2, j_2} p_2(y_{i_2}^{j_2}) \Delta y_{i_2}^{j_2}(m_{i_1}) \approx \rho P l_*^2(\mu^0) / [3\sigma r_*(\mu^1)]$$

where

$$P = \sin\varphi_0 \int_0^b p_1(x)dx \int_0^l p_2(y)dy$$

is the lifting force. Let l_0 be the value of $l_*(\mu^0)$ when $m_{i_1} = \overline{m}_{i_1} (i_1 = 1, \ldots, \overline{m})$, and let r_0 be the value of $r_*(\mu^1)$ when

$$m_{i(2)}^{j(1)} = m_{i_1}^{j_1} \quad (i_1 = 1, \ldots, \overline{m}; \; i_2 = 1, \ldots, \overline{m}_{i_1}; \; j_1 = 0, 1).$$

We then have the estimate

$$\rho P l_0^2 / (3\sigma r_0) \tag{3.11}$$

for the total mass of the zeroth-order spines (for the chosen values of the variables). We similarly obtain the estimate

$$\rho P l_1^2 / [3\sigma f(l)] \tag{3.12}$$

for the total mass of the first-order spines, where l_1 is the longest length of the first-order spines. Finally, we can similarly derive an estimate for the total mass of the second-order spines

$$G^* = \frac{\rho \sin\varphi_0 p_2(y_2^*) l^3}{3\sigma f(y_1^*)} \int_0^b p_1(x)dx \tag{3.13}$$

where the numbers y_2^* and y_1^* are taken in the interval $[0, l]$ so that

$$\frac{p_2(y_2^*) l^3}{6f(y_1^*)} = \int_0^l \frac{dy}{f(y)} \int_y^l p_2(\bar{y})(\bar{y} - y)d\bar{y}$$

Introducing a number λ_p such that

$$p_2(y_2^*) l = \lambda_p \int_0^l p_2(y)dy$$

we can rewrite (3.13) in the form

$$G^* = \rho P \lambda_p l^2 / [3\sigma f(y_1^*)] \tag{3.14}$$

The original parameters are usually such that the sequence of estimates (3.14), (3.12) and (3.11) decreases rapidly. Because $r_0 \approx \delta$, one can take $n=1$, when δ is close to $f(l)$, i.e. construct the wing as a collection of first-order feathers without losing a significant amount of mass.

The case of a wing with a single surface was considered without taking gravity into account. If gravity has to be taken into account, this can be done with several iterations of the method of successive approximations. Every νth iteration ($\nu=1, 2, \ldots$) consists of solving the problem in which $p_2(y)$ is replaced by $p_2^{(\nu)}(y)=p_2(y)-g_{\nu-1}(y)$, and obtaining the weight distribution function over the wing surface in the form

$$-p_1(x)g_\nu(y) \quad (x\in[0,b],\ y\in[0,l])$$

where $g_0(y)\equiv 0$, and according to (3.11)–(3.13)

$$g_\nu(y)=\frac{\rho g}{3\sigma}\left[\left(\frac{l_0^2}{r_0}+\frac{l_1^2}{f(l)}\right)p_2^{(\nu)}(y)+\frac{6}{f(y)}\int_y^l p_2^{(\nu)}(\bar{y})(\bar{y}-y)d\bar{y}\right]$$

where g is the acceleration due to gravity.

4. A WING WITH TWO SURFACES

We shall describe the main features for a wing constructed from two surfaces. Because the upper and lower wing surfaces are usually similar in shape to the plane $z=\text{const}$, instead of the wing surfaces we shall consider the planes $z=h$ and $z=-h$, where $0<h<f(l)$. We shall assume that the pressure drop when passing through the upper wing surface at any point (x, y, z^0) of that surface is the same as in passing through the plane $z=h$ at the point (x, y, h), and is described by the product $p_1^0(x)p_2^0(y)$, where $x\in[0, b]$, $y\in[0, l]$; the pressure drop when passing through the lower wing surface at the point (x, y, z^1) is the same as in passing through the plane $z=-h$ at the point $(x, y, -h)$ and is equal to $p_1^1(x)p_2^1(y)$. Here p_1^0, p_2^0, p_1^1 and p_2^1 are known continuous functions and positive inside their intervals of definition. Suppose that the functions p_2^0 and p_2^1 or p_1^0 and p_1^1 are linearly dependent, so that

$$p_1^0(x)p_2^0(y)+p_1^1(x)p_2^1(y)=p_1(x)p_2(y)$$

where $p_1(x)$ and $p_2(y)$ are non-negative functions $x\in[0, b]$, $y\in[0, l]$. We will call the projection onto the plane $z=h$ (and similarly $z=-h$) the upper (and similarly the lower) wing (or subfeather) projection.

Using the preceding formalization, one can introduce variables describing various representations of the wing in the form of collections of nth order subfeathers. First of all one can introduce a variable m giving the number of nth order subfeathers, assuming that the upper (lower) wing projection decomposes into upper (lower) subfeather projections onto the planes (2.1) satisfying condition (2.2) for equal loading. The upper (lower) projection for any i_1th subfeather ($i_1=1, \ldots, m$) divides into two subprojections with numbers $j_1=0, 1$ of the lines of intersection with the $x=x_{i_1}^c(m)$ plane, in which the spine axis of the subfeather lies and which satisfies condition (2.3) for symmetric loading. Then, as in the earlier formalization, one can introduce variables ${}^0m_{i_1}$, ${}^0m_{i(2)}^{j(1)}$ etc. ($i_1=1, \ldots, m$; $i_2=1, \ldots, {}^0m_{i_1}$; $j_1=0, 1$), describing the decomposition of the j_1th subprojection of the upper projection of the i_1th subfeather into projections of feathers of order $k<n$, having first replaced $p_1(x)$ by $p_1^0(x)$ and $p_2(y)$ by $p_2^0(y)$. One can similarly introduce variables ${}^1m_{i_1}$, ${}^1m_{i(2)}^{j(1)}$ etc. ($i_1=1, \ldots, m$; $i_2=1, \ldots, {}^1m_{i_1}$; $j_1=0, 1$), describing the decomposition of the j_1th subprojection of the lower projection of the i_1th subfeather into projections of feathers of order $k<n$, replacing $p_1(x)$ by $p_1^1(x)$ and $p_2(y)$ by $p_2^1(y)$.

Using the previously proved Assertions 1 and 2, we can verify that to minimize the wing mass one should take $n=2$, assign the largest possible values to the variables m, ${}^0m_{i_1}$, ${}^1m_{i_1}$ ($i_1 = 1, \ldots, m$) and the lowest possible values to the variables ${}^0m_{i(2)}^{j(1)}$ ($i_2 = 1, \ldots, {}^0m_{i_1}$; $j_1 = 0, 1$) and ${}^1m_{i(2)}^{j(1)}$ ($i_2 = 1, \ldots, {}^1m_{i_1}$; $j_1 = 0, 1$). Obviously, for these variable values the total mass of the kth order spines has the previous estimate: (3.14) for $k=2$, (3.12) for $k=1$, and (3.11) for $k=0$. In particular, for the $k=1$ this follows because estimate (3.12) depends linearly on the lifting force P. Thus the mass of a wing with two surfaces has the same estimate as the mass of a wing with a single surface.

5. CONCLUDING REMARKS

The results of this paper agree with the general rule of the construction of a bird's wing. We note two such rules: (1) the low order of feathers out of which the wing is constructed, and (2) the small width of the feathers and, consequently, their large number. These rules appear in the construction of wings for different purposes: (1) "gliding" wings, intended to obtain lift without simultaneously developing thrust, and (2) "flapping" wings, intended to develop both lift and thrust. To judge from the shape of a bird's tail, one can construct a "gliding wing" with a small aspect ratio and, consequently, low mass.

We shall give an example of a mass estimate for the wing of an aircraft. Suppose the take-off mass is $G_0 = 150$ kg, the analytical load factor coefficient $n_p = 5$, $p_2(y) = \text{const}$, the fuselage width $b_0 = 0.7$ m, and the wing parameters are as follows: $\varphi_0 = \pi/2$, $b = 1.2$ m, $l = 1.8$ m, $f(y) = 10^{-2}(4 - 3.9y/l)$ (m), $\delta = f(l)$. Suppose the wing material is the alloy B95T for which $\sigma = 6.08 \times 10^8$ N/m^2 and $\rho = 2.85 \times 10^3$ kg/m^3.

The analytic load factor on each wing is estimated from the formula $P = n_p G_0 g/2$, where we take $g = 9.81$ m/sec^2. Then, using (3.13), we find the dominant component of the mass of the wing

$$G^* = \frac{\rho P}{\sigma l}\int_0^l \frac{(l-y)^2}{f(y)}\,dy \approx \frac{\rho P l^2}{2\sigma f(0)} = 0.7 \text{ kg}$$

If we take $p_1(x) = \text{const}$ and $w = 2, 5$, the additional component (3.12) gives 0.06 kg. The mass of the wings therefore constitutes about 1% of the take-off mass, with an aspect ratio of $\lambda = (2l + b_0)/b \approx 3.6$, while the relative thickness is $c_0 = 2f(0)/b \approx 0.067$.

If in this example we take $b = 0.85$ m and $l = 2.6$ m, retaining the values of the other parameters and the wing area, we obtain $G^* \approx 1.46$ kg, so that the mass of the wing is about 2% of the take-off mass, $\lambda \approx 7$ and $c_0 = 0.094$. For comparison, we note that for well-known light aircraft with similar n_p, λ and c_0, the wing mass constitutes about 10% of the take-off mass [2].

The difference in wing mass between the traditional construction and that proposed here can be explained by the fact that a normal wing is constructed as a single cantilevered plate or as a shell with a framework [3]. The way the moments change from section to section of a single cantilever is usually substantially different from that for a construction composed of adjacent elementary cantilevers under the same load. This, in particular, leads to a mass saving if a wing constructed from a single (say, first-order) feather is replaced by a wing of several narrower feathers of the same order with the same external geometrical spine characteristics and preserving the plan shape of the wing.

A practical implementation of a wing of the proposed construction, in which all spines have annular sections, is difficult. However, the difficulties can be reduced if the leading order spines are made with the same closed annular section, and the remaining spines with open sections. For example, a wing can be constructed from feathers (or subfeathers) of second order, where spines of order $k < 2$ have a rectangular section with height that varies along the spine axis, and bounded by the values $f(l)$ for $k = 1$, δ for $k = 0$. For cross-sections of this shape one can justify assertions similar to those proved and arrive at analogous conclusions about wing construction. In practice a feather (or subfeather) can be constructed by joining several sections, each of which reproduces the part of the feather (or subfeather) between two transverse spine cross-sections. These sections can be produced by preparing a tube of variable cross-section with attached plates and then working each plate to obtain the required spines.

A wing with a traditional external shape can be constructed in the form of a row of adjacent subfeathers (see Fig. 1b) each of which is cantilevered to the fuselage. The subfeather includes within itself a spine of annular section, with external radius varying along the spine axis in the same way as the half-thickness of the wing varies with respect to span, and with first- or zeroth-order feathers cantilevered to this spine, these feathers being obtained, for example, by machining plates attached to the spine. The first- or zeroth-order feathers attached to the spine of the subfeather together produce part of the upper and lower surfaces of the wing and form its profile.

In order to minimize the wing mass the number of feathers (or subfeathers) attached to the fuselage is best chosen to be large, because the width of each feather (or subfeather) and the load on it should be small. The position of the (sub)feather spine axis is chosen according to the symmetric load condition to be such that there are no twisting moments in the spine. Hence, when there is a random variation in the aerodynamic load on the wing, the (sub)feather spine should not perform bending–twisting oscillations which can occur in a traditionally constructed wing and lead to its destruction [3]. Purely bending (sub)feather oscillations, which are possible under rapid random changes in the aerodynamic load, are damped because of the change in the lifting force which occurs because of the displacement of the construction elements when acted upon by elastic forces.

We add that according to the formal description the projection parameters of the (sub)feathers attached to the fuselage are chosen so as to ensure the same loading of the spines of those (sub)feathers, with the exterior radius of the section of each spine changing, by assumption, according to the same function $f(y)$. This ensures the same bending of the spines, taking into account that the inner radius of the spine section is chosen according to the strength condition. This ensures the preservation of the wing profile under bending.

If the formation of the wing profile requires one to use subfeathers whose outer spine radii vary along their axes according to different laws, then ensuring the same bending for the spines can require some excess in the mass of the wing. The smallest mass of such a wing, considering typical external profile shapes and load distributions, usually differs from the smallest wing mass when the external spine radii vary according to the same law in all the subfeathers, by no more than a factor of 1.5, for similar λ, c_0, l and P.

We note that the value of the parameter w, which sets a lower limit to the ratio of the width of the highest-order (sub)feather to the diameter of its largest external spine section, should be chosen considering not only the mass, but also some other efficiency characteristics. In particular, w can be selected by considering as well as the mass, the possibility of controlling the aircraft motion by changing the plan shape of the wing by rotating the (sub)feathers about parallel axes. It is desirable to perform this control in both wings in such a way that adjacent highest-order (sub)feathers partially overlap with partial superposition of the elements forming the wing profile, the control mechanism ensuring that (1) the wing area, and therefore the lifting force, vary so as to control the rolling moment, and (2) the point of application of the lifting force varies along the fuselage axis so as to control the pitching moment.

Note that when the proposed wing is used the undercarriage should be attached to the fuselage.

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Translated by R.L.Z